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Is the Urban World Small? The Evidence for Small World Structure in Urban Networks

Zachary Neal[1]

Abstract The initial definition of small world networks triggered a rush among network scientists, working in a variety of fields and with data from many different contexts, to identify and document empirical examples of small world networks. Researchers studying urban networks – networks of cities and networks in cities – have also participated in this exercise, but because their work took place in a variety of disciplines, no definitive answer has emerged to the question: Is the urban world small? I answer this question through a systematic review of the evidence for small world structure in 172 urban networks. I find that although authors often claim urban networks are small world (71.5%), such claims are rarely grounded in a formal index or guided by a specific decision rule, and may overestimate the ubiquity of this structure. However, existing indices of small worldliness offer promising options for summarizing the extent to which an urban network is small world. I conclude with recommendations that urban network researchers make use of these indices and begin conceptualizing small worldliness as a continuous, rather than binary, characteristic.

Keywords Small world · Network · Measurement

1 Introduction

An interest in networks has been present in *urban studies* for at least several decades (e.g. Haggett and Chorley 1969), and frequently network analyses within urban studies employ analytic techniques developed in related social science disciplines (e.g. centrality indices in sociology; Freeman 1978). However, a new brand of network analytic techniques have recently emerged, developed primarily in physical science disciplines such as physics, under the heading of *network science* (e.g. Newman 2010).

✉ Zachary Neal
zpneal@msu.edu

[1] Department of Psychology, Michigan State University, East Lansing, MI 48824, USA

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The techniques in the network science toolkit often focus on the topological structure of the graph as a mathematical object, rather than on the substantive content of the network as a conceptual object. They have found application in urban studies in a number of ways, including the detection of communities and the identification of specific degree distributions (e.g. Expert et al. 2011; Guimera et al. 2005; Neal 2014), however the integration of network science techniques and urban studies concepts has been uneven (Ducruet and Beauguitte 2014). This has been particularly true in the case of one network science focus: small world networks (Barthélemy 2011; Rozenblat and Melançon 2013).

In 1998, Watts & Strogatz published what would become a landmark paper in the emerging network science literature, defining a class of networks they called "small world." This name derived from a much earlier social psychology experiment – the small world experiment – which suggested that everyone in the world is connected to everyone else by just a few intermediary acquaintances (i.e. by "six degrees of separation"; Milgram 1967). The structure of networks belonging to this class helped explain how such a social phenomenon was possible. Watts and Strogatz (1998) initially showed that a small world structure could be observed in a network of Hollywood actor collaborations, the transmission line network of the U.S. power grid, and the neural network of the nematode *C. elegans*. This triggered a rush among network scientists, working in a variety of fields and with data from many different contexts, to identify and document empirical examples of small world networks (see Schnettler 2009, for a review). For example, Humphries and Gurney (2008) examined 33 different networks ranging from scientific coauthorship to linguistic associations to protein interactions. Researchers studying urban networks – both networks of cities and networks in cities – have also participated in this exercise, but because their work took place in a variety of disciplines, no definitive answer has emerged to the question: *Is the urban world small?*

The purpose of this paper is to seek an answer to this question through a systematic review of the evidence for small world structure in urban networks. To this end, I begin in section 2 by briefly reviewing what a small world network is, and why this class of network structure might be important in the urban context. In section 3, I describe a systematic search for all urban networks described in sufficient detail to evaluate whether they exhibit a small world structure, and review the three most widely used indices of small worldliness, which I then use to evaluate these networks in section 4. Finally, I conclude in section 5 with a discussion of these findings, focusing on recommendations for future research aimed at discerning whether an urban network is small world.

2 Background

2.1 What is a Small World Network?

The conception of small worldliness developed by Watts and Strogatz (1998) describes a class of networks that exist between the extremes of a highly ordered and highly disordered network (see Fig. 1). A highly ordered network such as a lattice is distinguished by two key structural characteristics. First, it has a relatively large clustering

	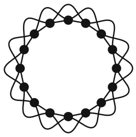	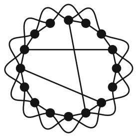	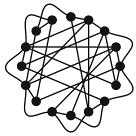
Network	Lattice, Ordered	Small World	Random, Disordered
Clustering Coefficient	High	High	Low
Mean Path Length	Long	Short	Short

Fig. 1 Lattice, Random, and Small World Networks

coefficient (*C*), which measures the extent to which two of a node's neighbors are connected to each other. For example, in the social context, this occurs when one's friends are also friends with one another. Second, it has a relatively large mean path length (*L*), which measures the average number of steps required to move from one node to another node along the shortest path. For example, in an infrastructure context, this might occur in a grid street network when a relatively large number of intersections must be crossed to move from one location to another.

A highly disordered network like a random graph is distinguished by the same two characteristics, but in the opposite direction. It has a relatively small clustering coefficient, as might be observed in the interactions among strangers at an airport: I interact with one stranger, then another, but these two likely do not interact with each other. Likewise, it has a relatively small mean path length, as might also be observed in stranger interactions in an airport: an infectious disease carried by a single traveler can rapidly spread to many others not just within the airport, but globally, in a short time (e.g. Balcan et al. 2009).

Intuitively, it seems that the midpoint between the extremes of ordered and disordered networks would be characterized by clustering coefficients and mean path lengths also at the midpoint of the extremes. However, Watts and Strogatz (1998) discovered a class of networks – small world networks – that have the relatively large clustering coefficient of an ordered network, but also the relatively small mean path length of a disordered network. They observed that such a structure occurs when a small number of edges are randomly re-wired in an ordered network. The first few re-wirings produce dramatic decreases in the network's mean path length, as previously distant parts of the network are brought closer together, but the large clustering coefficient remains mostly unchanged.

2.2 Why Do Small World Networks Matter for Urban Studies?

At first glance, small worldliness may seem to be a purely mathematical curiosity, however whether or not urban networks are small world may have important implications. Most abstractly, small worldliness has been hypothesized to be a universal feature of real-world networks, which hints at the existence of fundamental principles that organize how all networks – social, biological, material – form. Small worldliness has been documented in networks drawn from many different, and seemingly unrelated, contexts. For example, it is not immediately apparent why the structure of the neural network of a nematode should have anything in common with the structure of a power transmission network, but Watts and Strogatz (1998) found that they did. As evidence of the ubiquity of small worldliness emerges, it will be important to know whether this evidence extends to urban networks, and in turn whether urban network formation is shaped by forces common to other types of networks. If small worldliness is observed across a broad range of urban networks at different scales, subsequent work will be needed to understand why, but even simple spatial phenomena like Tobler's first law – that near things are more related than far things – may be a sufficient explanation. If most interactions, whether between cities in the world or people in a city, are localized, this would yield higher levels of clustering. However, the rare long-distance interactions in such cases – between two cities in different world regions or between two people in different neighborhoods – would be sufficient to yield a small mean path length.

More concretely, however, networks with small world structures can have practical advantages in several urban domains at different scales: infrastructure, social relations, and economic development (Rozenblat and Melançon 2013). An infrastructure network – whether streets, or public transit, or electricity – describes how a commodity is transmitted from one location to another via conduits that pass through intermediary locations. For example, a car is transmitted via streets that pass through intermediary intersections. When an infrastructure network has a small world structure, it is composed of densely connected clusters (giving rise to a large clustering coefficient), which are linked to one another by just a few bridging links (giving rise to a small mean path length). For example, local neighborhood streets have many intersections and routes to get from one nearby location to another, while whole neighborhoods are linked to each other by a freeway with no intersections and only one route (Marshall 2005). A similar phenomenon can be observed at the scale of entire city systems also: road networks within cities are well connected, while there are typically only a few routes between cities (c.f. Berry 1964). Such a structure offers a balance between two competing considerations. The local clusters provide redundancy: if a local neighborhood street is blocked, other routes are available. Conversely, the global bridges provide efficiency: the freeway facilitates longer distance and higher speed travel (Latora and Marchiori 2001).

Consider, next, the network of social relationships among people living in an urban neighborhood. When a social network has a small world structure, the dense clusters occur because social groups and cliques exist. For example, Belgian beer enthusiasts often know one another, while network scientists often know one another. In contrast, the between-cluster links occur because a few neighborhood residents are boundary spanners: one person may be both a Belgian beer enthusiast and a network scientist. Such a structure offers a balance between two competing forms of social capital (Neal 2015), which have variously been called bonding and bridging (Putnam 2001), closure and brokerage (Burt 2001), and strong and weak ties (Granovetter 1973). The within-group relationships provide sources of social support and a sense of belonging, while the between-group relationships provide access to useful information and resources that are not available in one's own social group.

Finally, consider a bipartite network of economic ties between firms and world cities, which can be projected to examine inter-firm or inter-city relations (Taylor and Derudder 2015). At the firm level, most interactions occur within sector, while a few interactions occur between sector, thereby potentially giving rise to a small world network. Similarly, at the city level, most interactions occur within-region, while a few interactions are between regions, again potentially giving rise to a small world network (Neal 2017a; Van Meeteren et al. 2016). Whether viewed at the firm or city level, a network with such a structure offers some notable economic advantages. Dense clusters of within-sector links among firms permit cooperation and agglomeration, while clusters of within-region links among cities facilitate regional coordination. Likewise, the bridging inter-sector and inter-regional links increase opportunities for innovation by ensuring the flow of novel information (Uzzi and Spiro 2005).

3 Methods

3.1 Systematic Literature Search

Figure 2 illustrates the process used to systematically search the literature for all urban networks described in sufficient detail to evaluate whether they exhibit a small world structure. First, using Google Scholar, I identified all materials written after 1998, when Watts and Strogatz (1998) initially defined small world networks, that contain the phrase "clustering coefficient," the phrase "path length," and one of more of the following phrases: "urban network(s)," "city network(s)," or "network(s) of cities." The former two phrases were required because these network characteristics must be reported to evaluate its small worldliness, while the latter phrases were used to restrict the scope of network content. It is important to note that there is limited consensus in

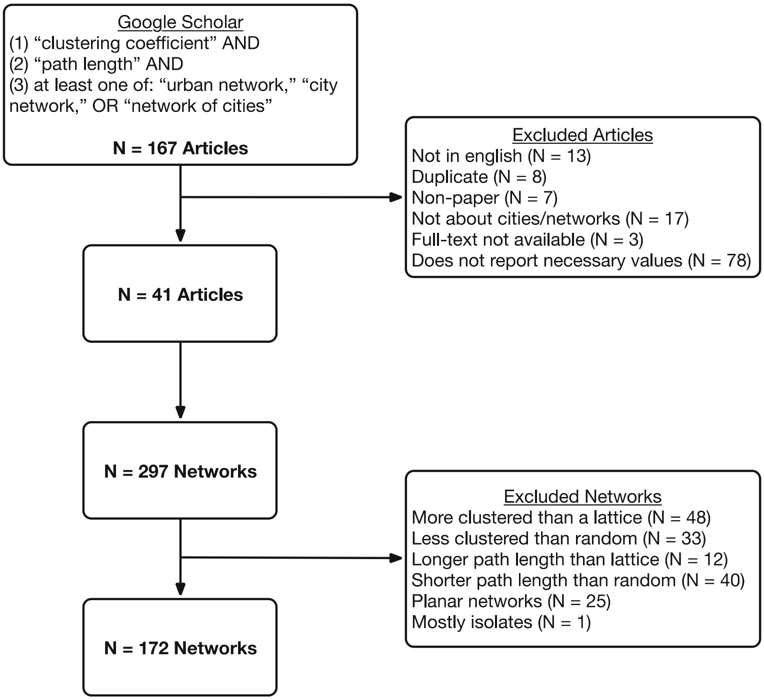

Fig. 2 Systematic literature search and data extraction

the urban studies literature about what constitutes an urban network; these terms have been applied to refer to many different types and scales of networks (Neal 2013). In this study, rather than adopt a specific definition of an urban network, I instead use a realist definition: a network is an urban network if the author calls it an urban network. This search yielded a pool of 167 articles and other materials. Next, I downloaded and skimmed each of these materials to determine whether each met the following inclusion criteria: (a) an article, book chapter, or dissertation, (b) for which the full text is available in English, (c) that reports on one or more urban networks, (d) for which the network's number of nodes (N), number of edges (E), clustering coefficient (C),[1] and mean path length (L) are provided or could be computed. This led to the exclusion of 126 (75.4%) materials, primarily because they did not report the necessary values.

From the remaining 41 materials (noted with an * in the references),[2] I extracted or computed the N, E, C, and L for each reported urban network, as well as the network's type and location, and whether the authors claimed the network displayed a small world structure. This yielded a sample of 297 networks. Because the Watts and Strogatz (1998) conception of small worldliness is defined only for networks whose C and L lie between the C and L in corresponding random (i.e. C_r and L_r) and lattice (i.e. C_l and L_l) networks, networks whose C and/or L lay outside these bounds were excluded from further analysis. Additionally, while planar networks are common in studies of urban infrastructure, they generally cannot have small world structures because they necessarily exhibit very little clustering, and therefore are excluded from further analysis.[3] Finally, one additional network, which was composed primarily of isolated nodes ($N = 325$, $E = 137$), was also excluded. This yielded a final sample of 172 networks for analysis.

3.2 Measuring Small Worldliness

A network has a small world structure when its clustering coefficient is similar to that of a corresponding ordered network (i.e. a lattice, $C \approx C_l$) and much larger than that of a corresponding disordered network (i.e. a random graph, $C >> C_r$), while its mean path length is similar to that of a disordered network (i.e. $L \approx L_r$) and much smaller than that of an ordered network (i.e. $L << L_l$). Three indices have been developed to quantify the extent to which a given network exhibits these properties (Neal 2017b), and are computed for each network in the sample to evaluate its small worldliness. The first

[1] It is important to observe that the clustering coefficient can be computed either as a global metric based on the number of closed triplets, or as the mean of a local clustering coefficient based on whether a node's neighbors are connected to each other. Frequently authors did not report which approach was used to compute the reported clustering coefficient. The Watts-Strogatz definition of small worldliness is grounded in the mean-local metric, and for the sake of analysis below, it is assumed this is what is reported by authors.

[2] The following papers are not cited elsewhere but were included in the review: Ansell et al. 2016; Brown et al. 2013; Chatterjee 2015; Chatterjee et al. 2016; Duan and Lu 2013, 2015; Ducruet and Notteboom 2012; Gao et al. 2007; Herrera-Yagüe et al. 2015; Ibanez 2015; Iovanovici et al. 2014; Jiang 2003; Johansson 2007; Kaltenbrunner et al. 2013; Liu 2007; Liu and Xiong 2016; Luo et al. 2016; Ma et al. 2011; Maniadakis and Varoutas 2012; Mansilla and Mendozas 2010; Mukherjee 2012; Porta et al. 2006; Saberi et al. 2017; Shi and Lu 2007; Sun 2013; Thakur 2012; Wang et al. 2014a, b; Xu et al. 2007; Yazdani and Jeffrey 2011; Zhang and Li 2012; Zhang et al. 2012.

[3] As expected, none of the planar networks in this sample were identified as having a small world structure. In principle, small worldliness could be investigated in planar networks by comparing their C and L values to those in reference *planar* random and lattice networks, however these values are not easy to compute.

such index was defined without a name by Kogut and Walker (2001), but later called the small-world quotient (Q) by Davis et al. (2003) and Uzzi and Spiro (2005):

$$Q = \frac{C/C_r}{L/L_r} \tag{1}$$

Values of $Q > 1$ are typically interpreted as evidence of small worldliness because they indicate that the network's L is similar to that of a random network and/or that the network's C is larger than that of a random network.

To address some limitations of Q (e.g. that it compares the given network only to a random network), Telesford et al. (2011) defined a new index normalized by comparison to both a random network and lattice as:

$$\omega = \frac{L_r}{L} - \frac{C}{C_l} \tag{2}$$

$$\omega' = 1 - |\omega| \tag{3}$$

Values of ω range from −1 to 1, with values near 0 interpreted as evidence of small worldliness. A minor transformation yields ω′, which is easier to interpret and compare with other indices, because it ranges from 0 to 1, with larger values reflecting greater small worldliness. Throughout this paper, for the sake of comparability to the other indicies, I use ω′.

To address some limitations of ω′ (e.g. failure to consider the ranges of C and L between lattice and random graphs) Neal (2015) defined the Small World Index (*SWI*) normalized by both C and L in both a random network and a lattice as:

$$SWI = \frac{L - L_l}{L_r - L_l} \times \frac{C - C_r}{C_l - C_r} \tag{4}$$

Like ω′, values of *SWI* range from 0 to 1, with larger values reflecting greater small worldliness.

The computation of these indices requires not only the observed network's C and L, but also these values in reference random networks and lattices. Following the approach initially used by Watts and Strogatz (1998) in their definition of small world structure, and in the subsequent application of these indices (Neal 2017b), the random reference graph used here is an Erdös–Rényi random graph, and the lattice reference graph is a one-dimensional ring lattice, both with the same number of nodes and edges as the observed network. The values of C and L in these reference graphs are computed as:

$$C_r = \frac{k}{n} \tag{5}$$

$$C_l \frac{3(k-2)}{4(k-1)} \tag{6}$$

$$L_r = \frac{\ln n}{\ln k} \tag{7}$$

$$L_l \frac{n}{2k} \tag{8}$$

where n is the number of nodes and k is the mean degree.

Neal (2017b) reviewed and compared the performance of Q, ω, and *SWI* as indices of small worldliness. He concluded that "although Q has been the most widely adopted index of small worldliness, alternative indices may be preferable" (p. 41). Both ω and *SWI* performed better, but captured slightly different conceptions of small worldliness. Focusing on the balance of clustering and connectedness, ω captures the extent to which an observed network approaches the most small worldly network that is possible given N and E. In contrast, *SWI* focuses on the theoretically ideal small world network, and captures the extent to which an observed network satisfies all four conditions in the definition: $C \approx C_l$, $C >> C_r$, $L \approx L_r$, and $L << L_l$. In the analyses below, I compute all three indices to examine the extent to which each provides evidence of the small worldliness of urban networks, and how these different indices may influence such claims in the urban network literature.

4 Results

Table 1 reports the descriptive characteristics of the networks that authors claim to be small world ($N = 123$), as well as those that might be deemed small world by the ω ($N = 144$) and *SWI* ($N = 59$) indices when using a 0.5 threshold. Within each column, the asterisks indicate the results of statistical tests (t-tests for continue variables, Fisher's exact tests for categorical variables) of the difference between these networks and the rest of the sample. It is important to note that the set of networks not identified as small world by an author includes both cases where the author explicitly stated that the network was not small world, and cases where the author did not make any claims about the small worldliness of the network. Although this represents a somewhat ambiguous category, it is a useful reference category because if an author computed and reported a network's C and L, they are likely to have claimed the network was small world if they believed it was. Thus, the absence of a claim of small worldliness likely reflects the author's belief that the network is not small world. Indeed, explicit claims that a network was not small world were rare in these articles.

Of the 172 networks reviewed, authors claimed that 123 (71.5%) of them were small world. Compared to networks that authors *did not* claim to be small world, these appeared in materials published on average significantly earlier (in 2009 vs. 2014), and were significantly different with respect to both their type and location. For example, compared to what would be expected at random, authors were more likely to view airline and transit networks and networks in Europe as small world, but less likely to view utility (e.g. water, electricity) networks and networks in the Americas as small world. While these characteristics present a descriptive portrait of the urban networks for which small worldliness has been claimed, of particular interest are the three indices that are intended to measure small worldliness (Q, ω, and *SWI*). The networks claimed to be small world do not differ significantly on any of these indices from those not claimed to be small world, which calls into question whether they are indeed small world and how authors reached such a determination. Taking a closer look at each index provides some insight.

It is not surprising that Q does not effectively distinguish networks claimed to be small world. Humphries and Gurney (2008) showed that Q linearly scales with network size (N), leading Neal (2017b) to contend that Q is a poor metric of small worldliness

Table 1 Characteristics of networks, by evidence for small worldliness ($N = 172$)

	Author claims network to be small world ($N = 123$)	Network where ω > 0.5 ($N = 144$)	Network where SWI > 0.5 ($N = 59$)
Publication year	2009 (3.56)**	2011 (3.94)	2011 (3.01)
Number of nodes (N)[a]	2834 (5952)	2995 (7227)**	2503 (3075)
Number of edges (E)[a]	39,686 (122759)	20,984 (70839)**	40,292 (104557)
Density	0.025 (0.038)	0.024 (0.038)	0.031 (0.039)*
Clustering coefficient (C)	0.270 (0.219)	0.290 (0.207)**	0.484 (0.153)**
Mean path length (L)	8.587 (8.049)	9.257 (7.909)*	5.748 (5.898)**
Indices –			
Q	4571 (33525)	378 (3554)**	92 (124)
ω	0.694 (0.219)	–	0.806 (0.133)**
SWI	0.413 (0.252)	0.457 (0.242)**	–
Type –	**		**
Airline ($N = 9$)	8 (89%)	9 (100%)	7 (78%)
Street ($N = 62$)	42 (68%)	54 (87%)	32 (52%)
Transit ($N = 71$)	60 (85%)	58 (82%)	12 (17%)
Utility (N = 5)	0 (0%)	5 (100%)	0 (0%)
Other Infrastructure ($N = 4$)	2 (50%)	3 (75%)	3 (75%)
Social ($N = 21$)	11 (52%)	15 (71%)	5 (24%)
Location –	**		**
Africa (N = 4)	4 (100%)	3 (75%)	1 (25%)
Americas ($N = 53$)	27 (51%)	46 (87%)	29 (55%)
Asia ($N = 46$)	32 (70%)	39 (85%)	14 (30%)
Australia (N = 5)	4 (80%)	4 (80%)	1 (20%)
Europe ($N = 58$)	52 (90%)	46 (79%)	8 (14%)
Global (N = 6)	4 (66%)	6 (100%)	6 (100%)

* $p < 0.05$; ** $p < 0.01$ (t-test or Fisher's exact test of difference compared to other networks)

[a] These values exclude three networks with $N > 1{,}000{,}000$

because all large networks will have $Q > 1$, and very large networks will have $Q >> 1$. Figure 3 shows the relationship between Q and N for all the networks included in this study, and confirms the linear scaling relationship ($\rho = 0.758$, $p < 0.001$). As networks increase in size, their Q increases proportionally, demonstrating that Q is not a measure of small worldliness, but instead is primarily measure of network size.

Unlike Q, Neal (2017b) found that ω and *SWI* can be suitable indices, but measure slightly different conceptions of small worldliness. Both indices view small worldliness as a matter of degree, but a threshold can be applied to make decisions about whether to consider a network small world. Because they both range from 0 (not small world) to 1 (maximally small world), a threshold of 0.5 can be used to identify networks that are, from the perspective of each index, more small world than not. The last two columns of Table 1 summarize the networks that would be identified as small world using this approach. Consistent with their differing operationalizations, ω is more liberal, identifying 83.7% of networks as small world, while *SWI* is more conservative, identifying

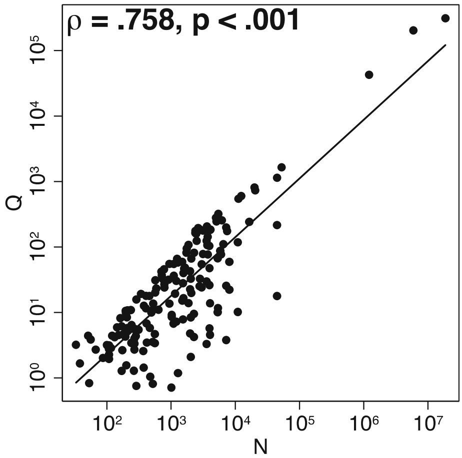

Fig. 3 Relationship between Q and N

only 34.3% as small world. Unlike the approach that has been used by authors to make decisions about networks' small worldliness, these indices are no more likely to identify networks as small world if they were published recently or several years ago. However, the networks they identify as small world do have statistically significantly different clustering coefficients and mean path lengths compared to non-small world networks, as would be expected given the definition of small worldliness.

The results in Table 1 suggest that authors have used ambiguous and potentially misleading strategies for identifying small worldliness that are more associated with the date of publication than with the network's structural properties. However, because the two available indices – ω and *SWI* – also differ on which networks are small world, how can these authors' claims be evaluated? Figure 4 plots the ω and *SWI* for all 123 urban networks authors claimed to be small world, and thus offers a way to explore discrepancies among claims of small worldliness. As expected for two indices designed to capture different understandings of the same concept, they are significantly positively correlated (ρ = 0.412, $p < 0.001$), but not perfectly so. When both indices are high, as in the case of the dots labeled A and B, they provide consistent evidence of small worldliness, and support the claims made by the authors writing about these networks. However, this plot also shows two types of disagreement. First, in some cases, an author's claim about a network's small worldliness disagrees with these two indices, which are consistent with each other (see dot C). Second, in other cases, the indices disagree with each other and yield conflicting evidence about an author's claim of a network's small worldliness (see dot D).

Table 2 reports the details of the urban networks that correspond to these four labeled dots in Fig. 4, and thus provide an opportunity to evaluate the performance of these indices and the accuracy of the authors' claims, relative to Watts and Strogatz's (1998) initial definition of small worldliness. Network A is a relatively large ($N = 3712$), global air transport network which Lordan et al. (2014) claimed to be small world. All three indices are consistent with each other, and with the authors' claim: $Q = 142$, $\omega = 0.876$, and $SWI = 0.923$. Close inspection of the network's C and L also suggest that the indices and authors' claim is accurate: C (0.64) is almost the same as that of a lattice

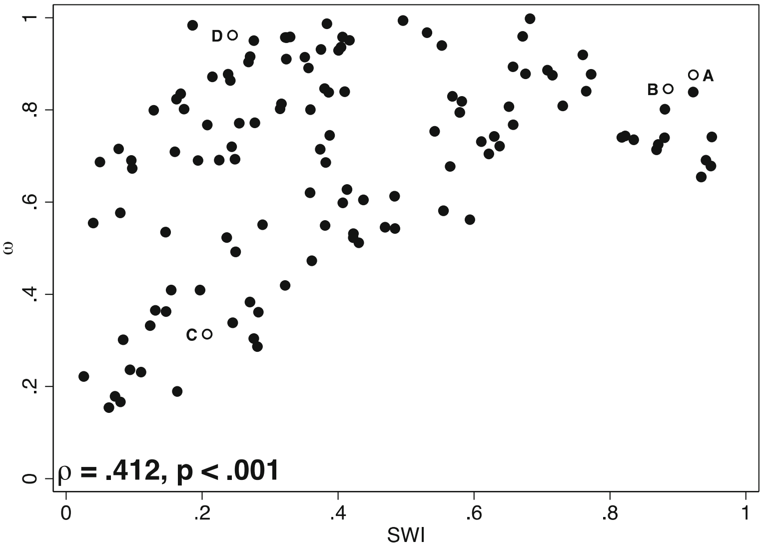

Fig. 4 Relationship between ω and SWI for networks claimed to be small world

($C_l = 0.689$), while L (3.94) is almost the same as that of a random graph ($L_r = 3.168$). This is an unambiguously small world network.

Network B is a much smaller ($N = 144$) air transport network focused only on airports in China, but which Wang et al. (2011) also claimed to be small world. Again,

Table 2 Characteristics of selected networks claimed to be small world

Network	A	B	C	D
Citation	Lordan et al. (2014)	Wang et al. (2011)	Jiang and Claramunt (2004)	Sienkiewicz and Hołyst (2005)
Type	Air Transport	Air Transport	Street	Bus Transit
Location	Global	China	San Francisco, USA	Gorzów Wielkopolski, Poland
N	3712	144	637	269
E	24,848	1018	2389	334
C	0.64	0.69	0.142	0.082
C_r	0.004	0.098	0.012	0.009
C_l	0.689	0.693	0.635	0.243
L	3.94	2.23	3.52	16.41
L_r	3.168	1.876	3.204	6.160
L_l	138.632	5.092	42.467	54.234
Q	142.687	5.912	10.980	3.339
ω	0.876	0.846	0.313	0.962
SWI	0.923	0.886	0.207	0.245

Key: C = Clustering coefficient, L = Mean path length, Subscript r = in an Erdös–Rényi random graph with the same size and density, Subscript l = in a one-dimensional ring lattice with the same size and density all three indices are consistent with each other, and with the authors' claim: $Q = 5.912$, $\omega = 0.846$, and $SWI = 0.886$. Moreover, inspection of the network's C and L confirm the accuracy of the indices and authors' claim: C (0.69) is almost the same as that of a lattice ($C_l = 0.693$), while L (2.23) is near that of a random graph ($L_r = 1.876$). Like network A, this is an unambiguously small world network. These two cases serve to illustrate how indices of small worldliness are intended to work, and highlight the repeated empirical finding that air transport networks are often small world (see also Guimera et al. 2005; Neal 2014; Segura Revilla 2011; Wijdeveld 2015). However, they also illustrate the problem with Q as an index of small worldliness: although inspection of C and L provides similar evidence of small worldliness for both networks, Q is dramatically larger for network A than network B, simply because network A is larger.

Network C is the street network of San Francisco, which Jiang and Claramunt (2004) claim is small world. This claim is supported by Q (10.98), and indeed they substantiate their claim by comparing the network's C and L to values in a random graph, thus implicitly relying on Q. However, both ω (0.313) and SWI (0.207) suggest that the network is not (or, at least, not particularly) small world. Inspection of the network's C and L help explain what is happening. Consistent with the definition of small worldliness, the network's L (3.52) is very similar to that of a random graph (3.204). However, although the authors are correct in observing that the network's C (0.142) is much larger than that in a random graph (0.012), it still does not come close to that in a lattice (0.635). Thus, this network has one characteristic of small worldliness (i.e. a small L), but not the other (i.e. a large C). This illustrates a case where Q led to an inaccurate claim of small worldliness, and where ω and SWI both more accurately capture the network's structure as not small world.

Finally, network D is the bus transit network of Gorzów Wielkopolski in Poland, which Sienkiewicz and Hołyst (2005) claim to be small world, but do not offer a rationale for this claim. Although their claim is supported by ω (0.962), it is not supported by SWI (0.245), and thus this illustrates a case where indices of small worldliness are inconsistent with each other. Again, inspection of the network's C and L, relative to a reference lattice and random graph explain why. Looking first at the network's L (16.41), ω focuses on the fact that it is just 2.66 times larger than L_r (6.16), while SWI focuses on the fact that it is 78% of the way from a transition from L_l (54.234) to L_r. Here, the two indices largely agree with each other: the network's L is substantially more similar to a random graph than a lattice, which is a characteristic of small worldliness. However, looking at the network's C (0.082), ω focuses on the fact that it is just 2.96 times larger than C_l (0.243), while in contrast SWI focuses on the fact that it is only 31% of the way from a transition from C_r (0.009) to C_l. Here, the two indices disagree with each other: ω focuses on showing that C is similar to a lattice, but SWI reminds us that it is nonetheless still more similar to a random graph, which is not a characteristic of small worldliness. In this case, the authors' claim that the network is small world is not wrong *per se*, but requires a rationale, and in particular a commitment to a specific conception of small worldliness.

5 Discussion

This systematic review of published urban networks has revealed a number of patterns in the evidence for small worldliness in the structure of urban networks. Authors' claims of small worldliness were relatively common immediately after Watts and Strogatz (1998) initially defined the property, but have since become less common. This may represent a waning interest in the identification of small world networks generally, and by extension among urban network researchers, but it also may suggest that authors have become more cautious in making claims about small worldliness. Nonetheless, claims of small worldliness in the urban network literature are often not grounded in a formal index or guided by a specific decision rule, but instead rely on "eyeballing" whether the observed network's C and L seem similar to those in appropriate reference networks. Perhaps for this reason, at least some of the claims that an urban network is small world may be inaccurate.

As Fig. 4 illustrates, 17.8% (22 of 123) claims of small worldliness are inconsistent with both available indices, while many more such claims are supported by only one index. Indeed, ω and *SWI* are consistent in providing support for only about 31.7% (39 of 123) of all authors' claims of small worldliness. These findings do not allow a definitive determination of how many, or which kind of, urban networks are small world. However, despite claims that small worldliness "is probably generic for many large, sparse networks" (Watts and Strogatz 1998, p. 441) and that "small worlds have been found to organize a remarkable diversity of systems" (Uzzi and Spiro 2005, p. 448), they do suggest that small worldliness is not a universal characteristic of urban networks, and that its ubiquity among urban networks has been overstated.

Taken together, these findings inform two broad suggestions for future research on small worldliness in urban networks. First, if a network is claimed to be small world, one or more indices should be computed and reported, together with the key structural characteristics necessary to verify its value (N, E, C, and L). Both ω and *SWI* appear to be reasonable alternatives, but the choice between indices should be informed by the researcher's specific conception of small worldliness: ω for optimizing the balance clustering against connectedness, and *SWI* for matching a theoretical ideal. In the absence of a specific conception, it may be appropriate to compute both, exercising caution if they disagree, but confidently declaring the network small world (or not) if they agree. Second, all claims of small worldliness in the urban network literature have been treated as binary: a network either is, or is not, small world. However, it may be more useful to think of small worldliness as a continuous property: a network may be more or less small world. These indices offer a way of measuring this continuous property, and primarily differ in how they weight and normalize the network's clustering coefficient and mean path length.

These findings and recommendations should be viewed in light of some limitations of this study. First, as with any systematic review, there is a risk of coverage bias. For example, some materials were excluded from review because they were unavailable ($N = 3$) or unavailable in English ($N = 13$). This review also excludes urban networks discussed in materials not indexed by Google Scholar, and in materials that do not use any of the following phrases: "urban network," "city network," or "network of cities." To limit coverage bias, however, this review does include unpublished materials such as dissertations and conference presentations. Second, the discussion and analysis has focused narrowly on the Watts and Strogatz (1998) model of small worlds, which views small worldliness as occurring during a transition between order (operationalized as a one-dimensional ring lattice) and disorder (operationalized as an Erdös–Rényi random graph) in non-planar graphs. This is the most widely adopted model of small worlds, and appears to be the one adopted in all articles included in this review, but it may not be the most suitable for spatially embedded networks (Barthélemy 2011; Rozenblat and Melançon 2013).

These findings and recommendations also highlight some avenues for future research on small worldliness in urban networks. First, although network type and location appear to be associated with claims of small worldliness, this review has not attempted to identify precisely which network operationalizations are more likely to yield small world structures. For example, road and transit networks can be represented as graphs in L-space (e.g. bus stops linked by direct service), B-space (e.g. bus stop-by-route bipartite), P-space (e.g. bus stop projection of bipartite), and C-space (e.g. route projection of bipartite; Von Ferber et al. 2009). Future studies may investigate whether the small worldliness of an urban network rests on how the network is operationalized. Second, although the Watts and Strogatz (1998) model is widely used, future studies may explore the use of spatial constraints (e.g. planar graphs) in the construction of reference graphs used to decide whether a spatially embedded network is small world (Ducruet and Beauguitte 2014; Erath et al. 2009). Finally, because small worldliness does not appear to be a universal characteristic of urban networks, future research should consider what processes might lead to small worldliness in urban networks, and under what circumstances small worldliness in urban networks is substantively meaningful or practically significant.

Since the initial definition of small world networks in 1998, network scientists working in many domains have sought to identify and document instances of this structure. The first instance of such an exercise in the domain of urban networks occurred in 2003, and subsequently many claims of small worldliness in urban networks have been advanced. This systematic review has aimed to review these claims and evaluate the evidence provided for them. I find that although small worldliness is far from universal, some urban networks do have a small world structure. However, the evidence supporting claims of small worldliness, and the methods used to measure it, in this literature has been limited. Fortunately, two indices – ω and *SWI* – offer promising options for summarizing the extent to which an urban network is small world, which in the future should be viewed as a continuous rather than binary characteristic of a network's structure.

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